Some Developments in the Tagged Signal Model

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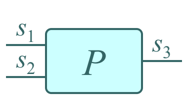
6th Biennial Ptolemy Miniconference

Berkeley, CA
May 12, 2005

The Tagged Signal Model

- A set of *tags* T, e.g. $T = [0, \infty)$
- A set of *values* V, e.g. $V = \mathbf{N}$
- An *event* e is a pair of a tag and a value: $e = (t, v)$
- A *signal* s is a set of events, e.g. $clock_1 = \{(0.0, 1), (1.0, 1), (2.0, 1), \ldots\}$
- A *process* P is a relation on signals



$$P = \{ (s_1, s_2, s_3) \mid s_1 + s_2 - s_3 = 0 \}$$

"A Framework for Comparing Models of Computation,"
Lee and Sangiovanni-Vincentelli, 1998

Signals and Processes

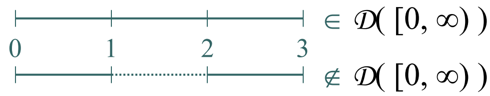
	Signals	Processes
Physics	Velocities, Accelerations, and Forces	Newton's Laws
Electrical Engineering	Voltages and Currents	Resistors and Capacitors, Kirchhoff's Laws
Computer Science	Streams	Dataflow Processes

Approach

- Study the mathematical structure of signal sets
  - Partial order/CPO, topological/metric space, algebra
- Study the properties of processes as relations/functions on signals
  - Continuity
  - Causality
  - Composition
- From the declarative to the imperative

Signals

- Let T, a poset, be the set of all tags. Let $\mathcal{D}(T)$ be the set of down-sets of T.



- A signal is a function from a down-set $D \in \mathcal{D}(T)$ to some value set V,

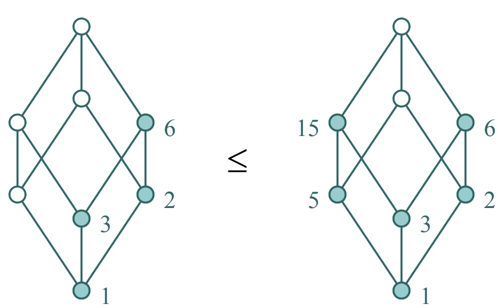
$$signal: D \to V$$

- Let $\mathcal{S}(T, V)$ be the set of all signals from down-sets of T to V.

Prefix Order on Signals

- A signal $s_1: D_1 \to V$ is a prefix of $s_2: D_2 \to V$, denoted $s_1 \leq s_2$, if and only if

$$D_1 \subseteq D_2, \text{ and } s_1(t) = s_2(t), \forall t \in D_1$$

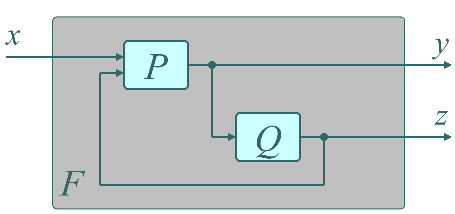


Prefix Order − Properties

- For any poset T of tags and set V of values, $\mathcal{S}(T, V)$ with the prefix order is
  - a poset
  - a CPO
  - a complete lower semilattice (i.e. any subset of signals have a "longest" common prefix)

Tagged Process Networks

- A direct generalization of Kahn process networks

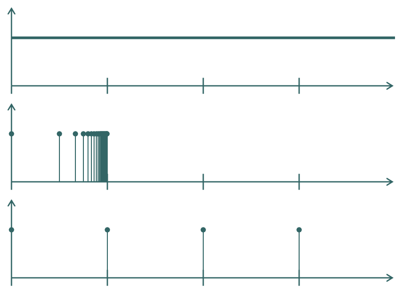


$(y, z) = F(x)$
where (y, z) is the least solution of the equations
$y = P(x, z)$
$z = Q(y)$

- If processes P and Q are Scott-continuous, then F is Scott-continuous.

Timed Signals

- Let $T = [0, \infty)$, and $V_\varepsilon = V \cup \{\varepsilon\}$, where ε represents the absence of value, $\mathcal{S}(T, V_\varepsilon)$ is the set of timed signals.

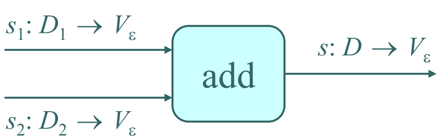


$s(t) = 1$

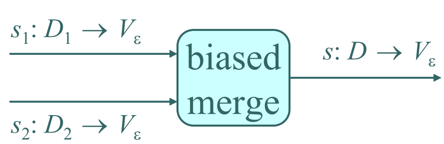
$s(1\text{-}1/k) = 1, k = 1, 2, \ldots$

$s(k) = 1, k = 0, 1, 2, \ldots$

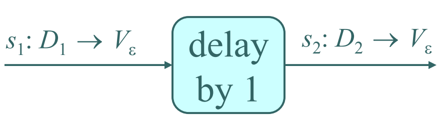
Timed Processes



$D = D_1 \cap D_2$
$s(t) = s_1(t) +_\varepsilon s_2(t)$

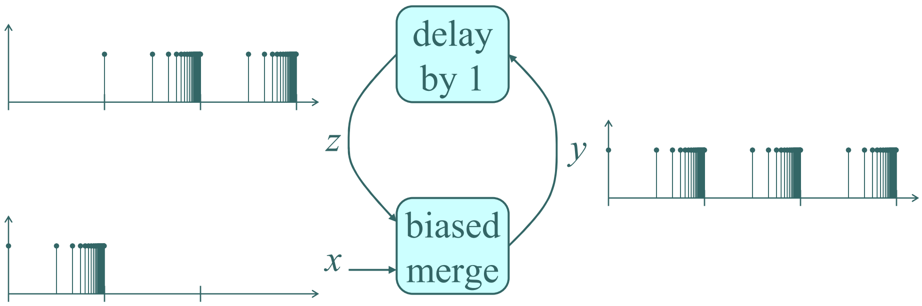


$D = D_1 \cap D_2$
$s(t) = s_1(t)$, when $s_1(t) \in V$
$s_2(t)$, otherwise

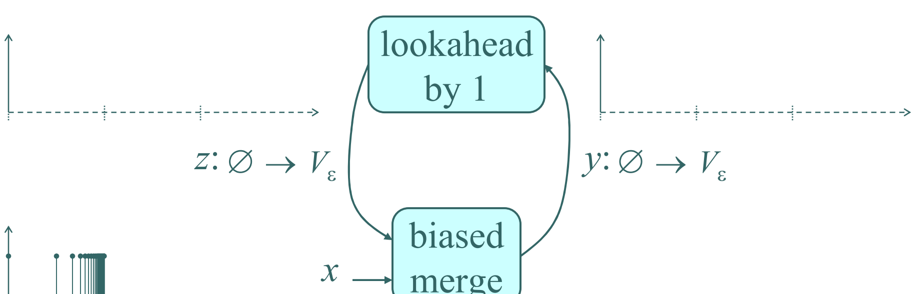


$D_2 = D_1 \oplus \{1\} \cup [0, 1)$
$s_2(t) = s_1(t - 1)$, when $t \geq 1$
ε, when $t \in [0, 1)$

A Timed Process Network



A Non-Causal Process in the Network



Causality

- A timed process P is causal if
  - It is monotonic, i.e. for all s_1, s_2

    $$s_1 \le s_2 \;\Rightarrow\; P(s_1) \le P(s_2)$$
  - For all $s: D_1 \to V_1$, $P(s): D_2 \to V_2$

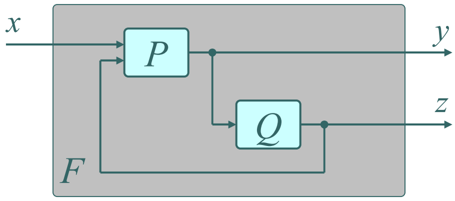
    $$D_1 \subseteq D_2$$
- A timed process P is strictly causal if it is monotonic, and
  - For all $s: D_1 \to V_1$, $P(s): D_2 \to V_2$

    $$D_1 \subset D_2 \;\text{ or }\; D_2 = [0, \infty)$$

Causality and Continuity

- Neither implies the other.
- A process may be continuous but not causal, e.g. “lookahead by 1”.
- A process may be causal but not continuous, e.g. one that produces an output event after counting an infinite number of input events.

Causal Timed Process Networks



$(y, z) = F(x)$

where (y, z) is the least solution of the equations

$y = P(x, z)$

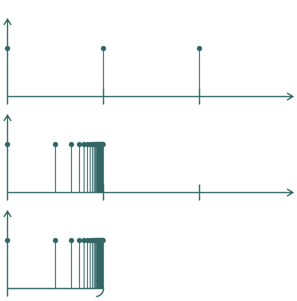
$z = Q(y)$

- If processes P and Q are causal and continuous, and at least one of them is strictly causal, then F is causal and continuous.

Discrete Event Signals

- A timed signal $s: D \to V_\varepsilon$ is a discrete event signal if for all $t \in D$

$$s^{-1}(V) \cap [0, t] \text{ is a finite set}$$



$\text{dom}(s) = [0, \infty)$ — DE, Non-Zeno

$s(k) = 1, k = 0, 1, 2, \ldots$

$\text{dom}(s) = [0, \infty)$ — Not DE

$s(1\text{-}1/k) = 1, k = 1, 2, \ldots$

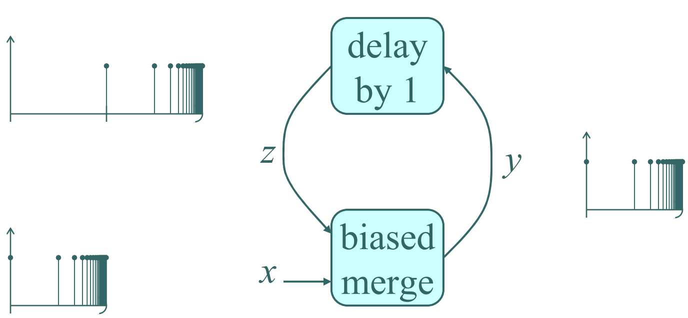
$\text{dom}(s) = [0, 1)$ — DE, Zeno

$s(1\text{-}1/k) = 1, k = 1, 2, \ldots$

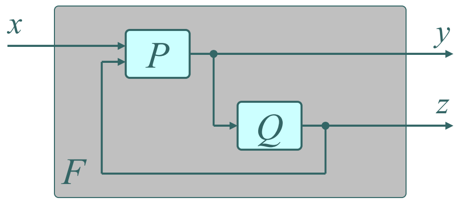
Discrete Event Signals - Properties

- For $T = [0, \infty)$ and any set V of values, the set of all discrete event signals with the prefix order is
  - a poset
  - a CPO
  - a complete lower semilattice (i.e. any subset of signals have a "longest" common prefix)

A Discrete Event Process Network



A Sufficient Condition for Non-Zeno Composition



$(y, z) = F(x)$

where (y, z) is the least solution of the equations

$y = P(x, z)$

$z = Q(y)$

- If processes P and Q are discrete, causal and continuous, and at least one of them is strictly causal, then F is discrete, causal and continuous.
- F is non-Zeno in the sense that if x is non-Zeno, $F(x)$ is non-Zeno.

Conclusions

- Progress in developing the foundation of the tagged signal model
- Extend Kahn process networks to tagged process networks
- Develop discrete event semantics as a special case of tagged process networks
- Develop a sufficient condition for the non-Zeno composition of discrete event processes