Multidimensional Dataflow

Ptolemy ILP Conference

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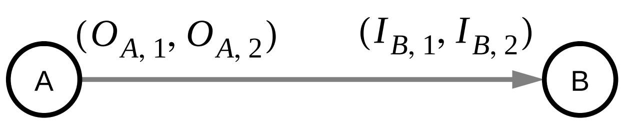
Multidimensional Dataflow Extension



Balance equations:

$$r_{A,1} O_{A,1} = r_{B,1} I_{B,1}$$

$$r_{A,2} O_{A,2} = r_{B,2} I_{B,2}$$

Solve for the smallest integers $r_{X,i}$, which then give the number of repetitions of actor X in dimension i.

Higher dimensionality follows similarly.

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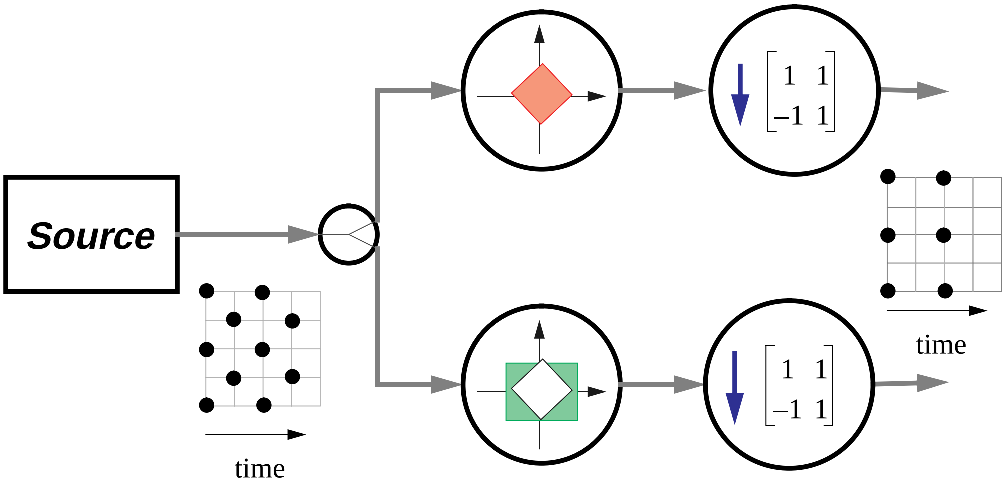
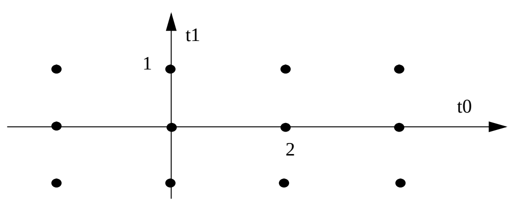
Generalization to Arbitrary Lattices

- **MDSDF handles only rectanglularly sampled signals.**
- **GMDSDF handles signals on arbitrary lattices,** ***without sacrificing compile-time schedulability.***



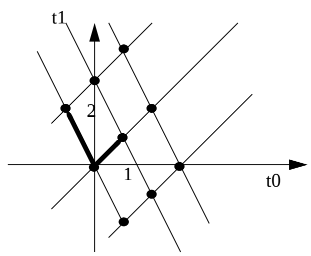
Non-rectangular Sampling

Rectangular sampling



$$V = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$$

Non-rectangular sampling

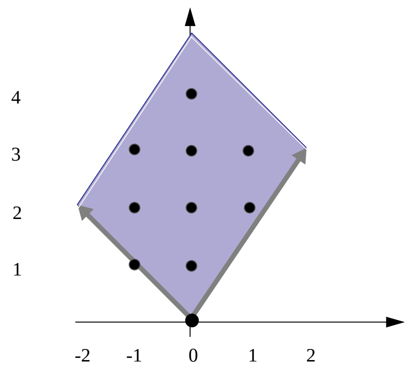


$$V = \begin{bmatrix} 1 & -1 \\ 1 & 2 \end{bmatrix}$$

Definition: The set of all sample points given by $\hat{t} = V\hat{n}$, $\hat{n} \in \aleph$ is called the ***lattice*** generated by V. It is denoted $LAT(V)$.

The Fundamental Parallelpiped

The ***fundamental parallelpiped***, denoted by $FPD(V)$, is the set of points given by Vx where $x = [x_1, x_2]^T$ with $0 \le x_1, x_2 < 1$.



$$L = \begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix},$$

Definition: The set of integer points in $FPD(V)$ is denoted as $N(V)$.

Lemma: $J(V) = |N(V)| = |det(V)|$ for an integer matrix V.

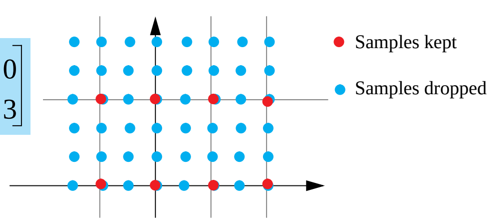
Multidimensional Decimators

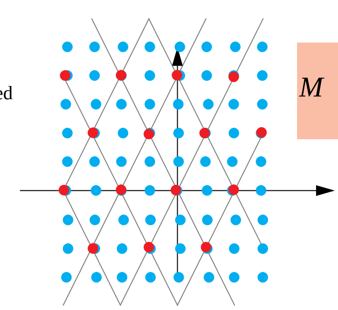
M-D decimation is given by the relationship:

$$y(\hat{n}) = x(\hat{n}), \hat{n} \in LAT(V_I M)$$

where x is defined on the points $V_I k$, V_I being the sampling matrix.

$$M = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$$

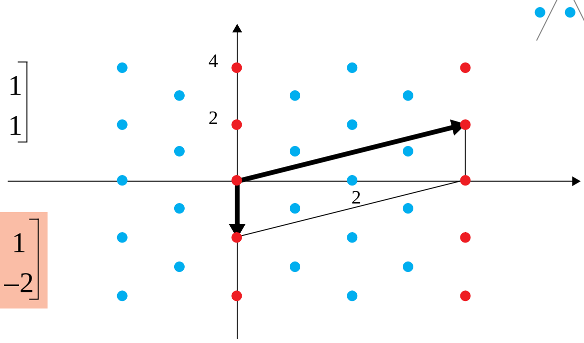




$$M = \begin{bmatrix} 1 & 1 \\ 2 & -2 \end{bmatrix}$$

$$V_I = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$$

$$M = \begin{bmatrix} 1 & 1 \\ 2 & -2 \end{bmatrix}$$



Decimation ratio:

$|det(M)|$

Multidimensional Expanders

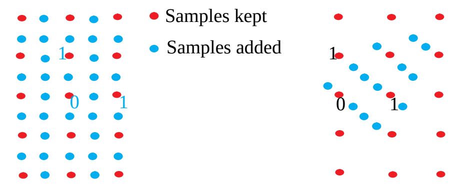
M-D expander:

$$y(n) = \begin{cases} x(n) & n \in LAT(V_I) \\ 0 & \text{otherwise} \end{cases} \forall n \in LAT(V_I L^{-1})$$

where x is defined at the points $V_I k$, V_I being the sampling matrix.

Rectangular expansion Non-rectangular expansion

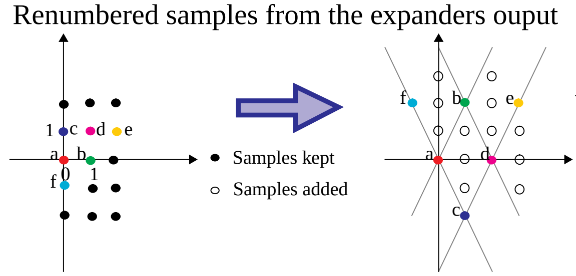
$L = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$



$L = \begin{bmatrix} 1 & 1 \\ 2 & -2 \end{bmatrix}$

Renumbered samples from the expanders ouput

$L = \begin{bmatrix} 1 & 1 \\ 2 & -2 \end{bmatrix}$



Expansion ratio:

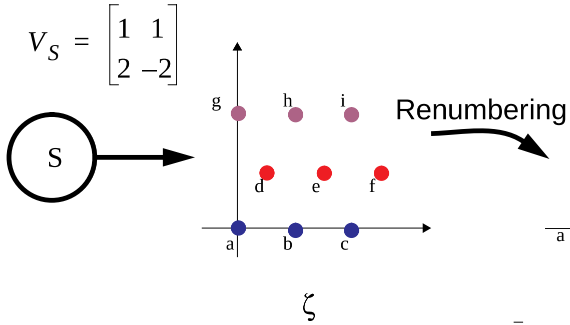
$|det(L)|$

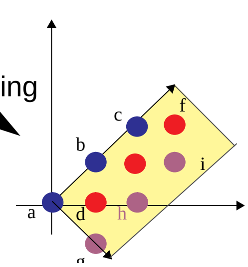
Genarlized MDSDF (GMDSDF): Sources

Definition: The **containability condition:** let X be a set of integer points in $\Re^m$. We say that X satisfies the *containability condition* if there exists an $m \times m$ matrix W such that $N(W) = X$.

Definition: We will assume that any source actor in the system produces data in the following manner. A source S will produce a set of samples ζ on each firing such that each sample in ζ will lie on the lattice $LAT(V_S)$. We assume that the renumbered set $\bar{\zeta}$ satisfies the containability condition.

$V_S = \begin{bmatrix} 1 & 1 \\ 2 & -2 \end{bmatrix}$ $Q = \begin{bmatrix} 3 & 1.5 \\ 3 & 1.5 \end{bmatrix}$



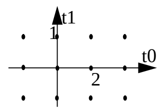


$$\bar{\zeta} = \{V_S^{-1} x : x \in \zeta\}, \bar{\zeta} = N(Q)$$

Concise Problem Statement

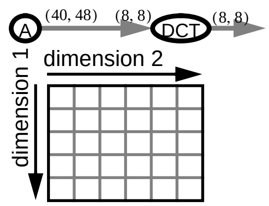
MDSDF

- **Rectangular lattice**



- **Regions of data produced = rectangular arrays**

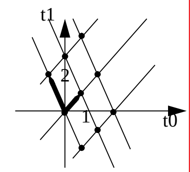
- **Rectangular arrays specified concisely by tuples of produced/ consumed.**



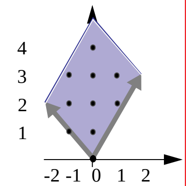
- **Coordinate axes for dataflow along arcs orthogonal to each other (x and y axes).**

GMDSDF

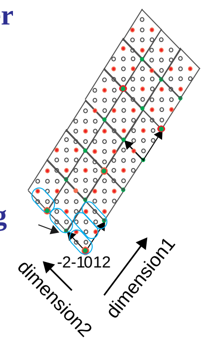
- **Arbitrary lattice**



- **Regions of data produced = parallelograms**



- **Parallelograms specified concisely as the set of integer points inside a support matrix.**

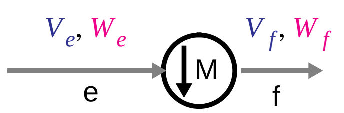


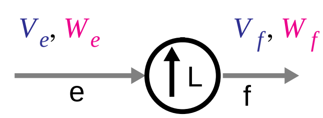
- **Coordinate axes for dataflow along arcs not necessarily othogonal.**

Support Matrices

Want to describe regions where the data is contained.

- **In MDSDF, these are ordinary arrays**
- **In the extension, these are *support matrices*.**





Theorem:

For the decimator,

$$V_f = V_e M \text{ and } W_f = M^{-1} W_e.$$

For the expander,

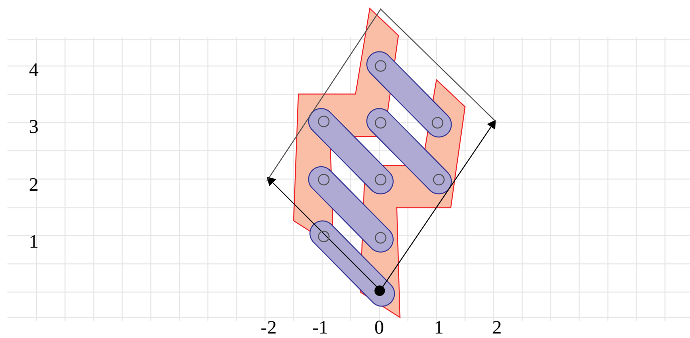
$$V_f = V_e L^{-1}, \text{ and } W_f = L W_e.$$

Semantics of GMDSDF

$$V_{SA} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad L = \begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix}, |L| = 5 \times 2 \qquad M = \begin{bmatrix} 1 & 1 \\ 2 & -2 \end{bmatrix}, |M| = 2 \times 2$$

$$W_{SA} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

A consumes (1,1) and produces (5,2).
B consumes (2,2) and produces (1,1) *on average*.
T consumes (1,1)



GMDSDF — Balance Equations

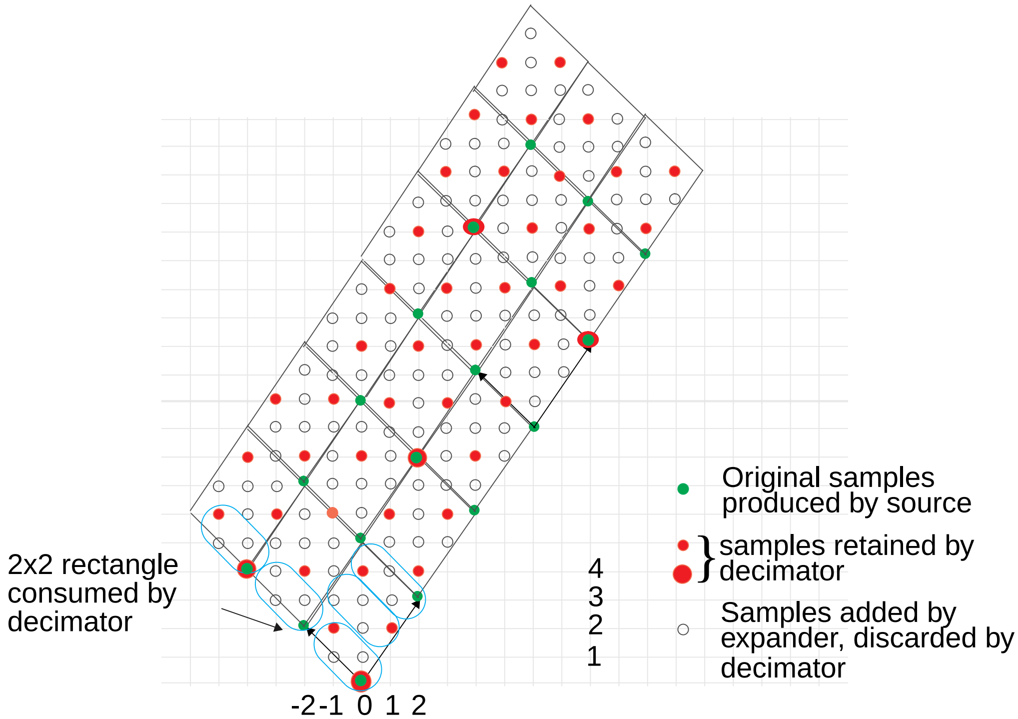
- **We don't know yet exactly how many samples on each firing the decimator will produce.**
- **Idea: *Assume* that it produces (1,1) and solve balance equations:**

$$\begin{matrix} 3r_{S,1} = 1r_{A,1} \\ 3r_{S,2} = 1r_{A,2} \end{matrix}, \quad \begin{matrix} 5r_{A,1} = 2r_{B,1} \\ 2r_{A,2} = 2r_{B,2} \end{matrix}, \quad \begin{matrix} r_{B,1} = r_{T,1} \\ r_{B,2} = r_{T,2} \end{matrix}$$

- **Solution:**

$$r_{S,1} = 2, r_{S,2} = 1$$
$$r_{A,1} = 6, r_{A,2} = 3$$
$$r_{B,1} = 15, r_{B,2} = 3$$
$$r_{T,1} = 15, r_{T,2} = 3$$

Dataspace on arc AB



Balance equations cont'd

Question: Have we really "balanced"?

No: by counting the number of samples that have been kept in the previous slide.

More systematically:

$$W_{SA} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} r_{S,1} & 0 \\ 0 & r_{S,2} \end{bmatrix} = \begin{bmatrix} 3r_{S,1} & 0 \\ 0 & 3r_{S,2} \end{bmatrix}$$

$$W_{AB} = LW_{SA} = \begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 3r_{S,1} & 0 \\ 0 & 3r_{S,2} \end{bmatrix} = \begin{bmatrix} 6r_{S,1} & -6r_{S,2} \\ 9r_{S,1} & 6r_{S,2} \end{bmatrix}$$

$$W_{BT} = M^{-1}W_{AB} = \frac{1}{4}\begin{bmatrix} 2 & 1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 6r_{S,1} & -6r_{S,2} \\ 9r_{S,1} & 6r_{S,2} \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 21r_{S,1} & -6r_{S,2} \\ 3r_{S,1} & -18r_{S,2} \end{bmatrix}$$

Balance equations cont'd

Want to know if

$$|N(W_{BT})| = \frac{|N(W_{AB})|}{|M|}$$

We have

$$|N(W_{AB})| = |det(W_{AB})| = 90r_{S,1}r_{S,2}$$

The right hand side becomes

$$\frac{90r_{S,1}r_{S,2}}{4} = \frac{45r_{S,1}r_{S,2}}{2}$$

Therefore, we need

$$r_{S,1}r_{S,2} = 2k \qquad k = 0, 1, 2, \ldots$$

The balance equations gave us $r_{S,1} = 2, r_{S,2} = 1$.

With these values, we get

$$W_{BT} = \begin{bmatrix} 21/2 & -3/2 \\ 3/2 & -9/2 \end{bmatrix}.$$

This matrix has 47 points inside its FPD (determined by drawing it out).

==> Balance equation solution is not quite right.

Augmented Balance Equations

To get the correct balance, take into account the constraint given by

$$|N(W_{BT})| = \frac{|N(W_{AB})|}{|M|}$$

Sufficiency: force W_{BT} to be an integer matrix.

$$\text{==>}\ r_{S,1} = 4k, k = 1, 2, \ldots$$

$$\text{==>}\ r_{S,2} = 2k, k = 1, 2, \ldots$$

Therefore,

$$r_{S,1} = 4, r_{S,2} = 2.$$

- **So decimator produces (1,1) on average but has cyclostatic behavior.**

Production sequence: 2,1,1,2,1,0,1,1,0,1,2,1,1,2,1,...

Theorem:

Always possible to solve these *augmented* balance equations.

Effect of Different Factorizations

Suppose we let $|det(M)| = 1 \times 4$ instead. Balance equations give:

$$r_{S,1} = 1, r_{S,2} = 2$$
$$r_{A,1} = 3, r_{A,2} = 6$$
$$r_{B,1} = 15, r_{B,2} = 3$$
$$r_{T,1} = 15, r_{T,2} = 3$$

Also,

$$W_{BT} = \begin{bmatrix} 21/4 & -3 \\ 3/4 & -9 \end{bmatrix}$$

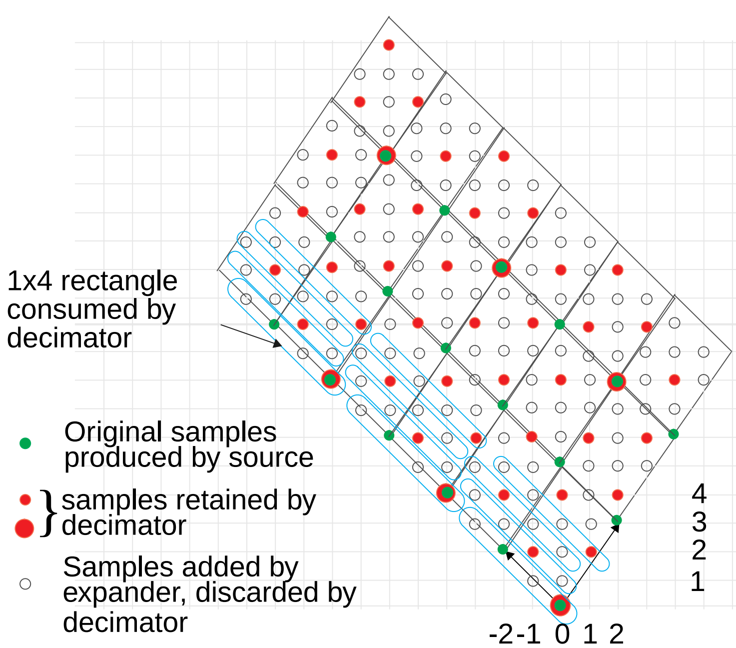
It turns out that

$$|N(W_{BT})| = 45$$

as required.

==> Lower number of overall repetitions with this factoring choice.

Dataspace on Arc AB

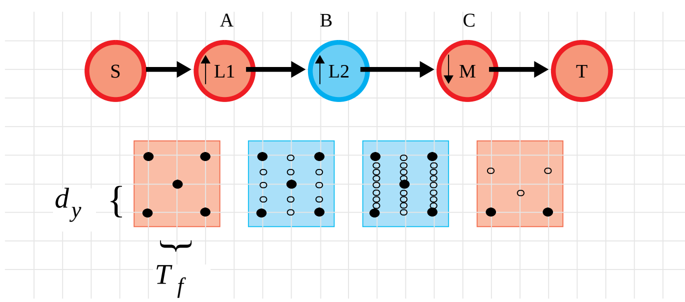


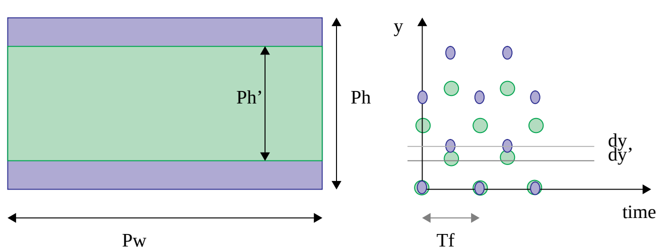
Summary of Extended Model

- **Each arc has associated with it a lattice-generating matrix, and a support matrix.**
- **The source actor for an arc establishes the ordering of the data on that arc.**
- **Expander: consumes (1,1) and produces $FPD(L)$, ordered as an (L_1, L_2) rectangle where $L_1 L_2 = |det(L)|$.**
- **Decimator: consumes an (M_1, M_2) rectangle, where $M_1 M_2 = |det(M)|$ and produces (1,1) on average.**
- **Write down balance equations.**
- **Additional equations for support matrices on decimator outputs.**
- **The above two sets are simultaneously solved to determine the smallest non-zero number of times each node is to be invoked in a periodic schedule.**
- **Actors are then scheduled as in SDF or MDSDF.**

Aspect Ratio Conversion

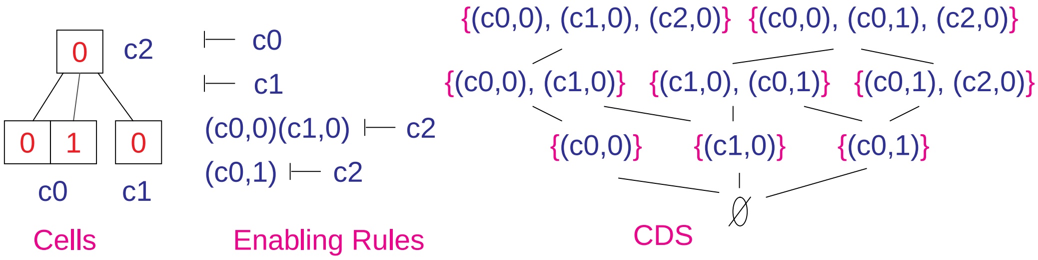
Format conversion of 2:1 interlaced video from 4/3 aspect ratio to 16/9 aspect ratio.





Concrete Data Structures

- "Cells" can have specific "Values"
- Enabling relationship says when a cell can be filled.
- "Cell" dependency partial order can be arbitrary
- Formalizes most forms of "real-world" data structures: lists, trees, arrays etc.
- Kahn-Plotkin sequential functions on CDS provide an elegant model of computation with many formal properties, like full abstraction.
- CDS approach has been mostly semantic; need to sort out operational issues (like scheduling).



Array-OL

- Array-oriented language developed at Thomson
- Graphical syntax for specifying "array access patterns"
  - In many multidimensional programs, manipulating data aligned in various dimensions is a challenge. For example: Transpose.
  - Patterns specified by "paving" and "tiling" relationships.
- Combine with MDSDF...