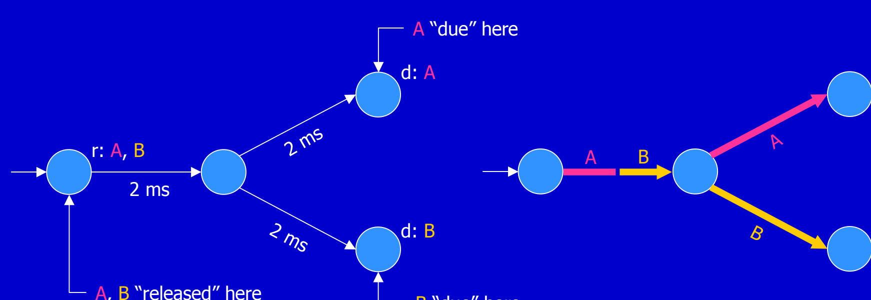
Conditional scheduling with varying deadlines

Ben Horowitz
bhorowit@cs.berkeley.edu

Which output: A or B?

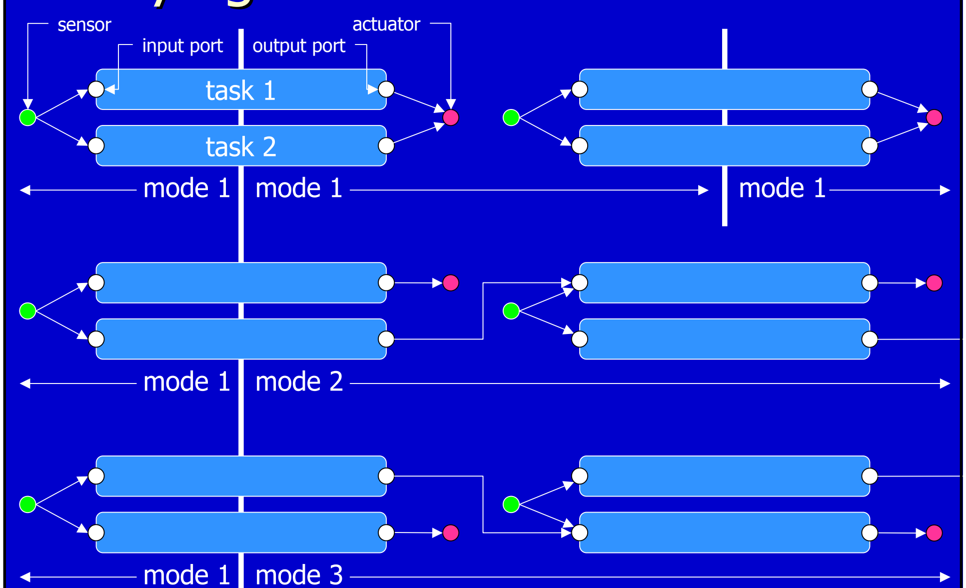
- A, B each require 3 milliseconds to compute.
- In 4 milliseconds, one will need to be output.
- Decision about which to output in 2 milliseconds.
- Speculatively start to compute both!



Varying deadlines in Giotto

- I first saw this problem when working on precedence-constrained Giotto scheduling.
- A task is invoked; when the task's actual deadline is depends on future mode changes.
- Following one set of mode changes, the task may have a 5ms deadline, say. Following another, the task may have a 10ms deadline.

Varying deadlines in Giotto



Conditional scheduling problem

Finite state machine:

- Set *Vertices* of vertices.
- Set *Edges* of edges.
- For each edge e a number $duration(e)$.
- Initial vertex v_0.

Workload:

- Set *Tasks* of tasks.
- For each $t \in Tasks$, a number $time(t)$.
- For each $v \in Vertices$, $release(v) \subseteq Tasks$.
- For each $v \in Vertices$, $due(v) \subseteq Tasks$.

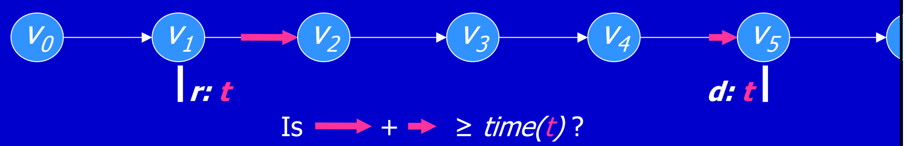
Game: scheduler vs. environment

- Let *Runs* = set of paths of length ≥ 2.
- Strategy is a function:

$$\sigma: Runs \times Tasks \to \Re$$

$$\Sigma_{t \in Tasks}\, \sigma((\ldots, v_i, v_{i+1}), t) \leq duration(v_i, v_{i+1})$$

When is a strategy winning?



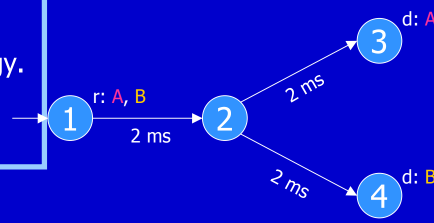
- Consider arbitrary run, position v_i.
- Consider arbitrary task t in $release(v_i)$.
- Find first subsequent v_j at which t is due.
- Let n = # of times t is released at/after v_i, before v_j.
- Strategy must allocate $n \times time(t)$ between v_i and v_j.

Related models

- [Baruah 1998a, 1998b]: Introduced conditional scheduling model.
  - Tasks have fixed deadlines.
  - EDF is optimal.
  - Question is: how to determine if demand exceeds processing time?
- [Chakraborty, Erlebach, and Thiele, 2001]: Hardness results and approximation algorithm to answer above question.
- Our model generalizes these:
  - Deadlines of tasks vary.
  - Extends to include precedence constraints.

Algorithm for strategy synthesis

1) Construct linear constraints on strategy.
2) Solve using linear programming.
   - A feasible sol'n is a winning strategy.
   - No feasible sol'n: no winning strategy.



- Interval constraints:
  - $\sigma((1, 2), A) + \sigma((1, 2), B) \leq 2$
  - $\sigma((1, 2, 3), A) + \sigma((1, 2, 3), B) \leq 2$
  - $((1, 2, 4), A) + \sigma((1, 2, 4), B) \leq 2$
- Task constraints:
  - $\sigma((1, 2), A) + \sigma((1, 2, 3), A) \geq 3$
  - $\sigma((1, 2), B) + \sigma((1, 2, 4), B) \geq 3$

Discrete-time conditional scheduling

- What if the scheduler can make decisions only at a restricted set of points? Switching triggered by, e.g., a timer interrupt.
- For simplicity suppose this set is the integers.
- **Theorem.** Deciding whether a discrete-time problem has a winning strategy is NP-hard.
- Under a reasonable definition of lateness, there is no 2-approximation algorithm unless P=NP.

Tree scheduling vs. DAG scheduling

- Our linear programming algorithm is polynomial-time only if *(Vertices, Edges)* is a tree.
- What if the graph is a directed acyclic graph (DAG)?
- **Theorem.** Determining whether a DAG problem has a winning strategy is coNP-hard.
- I believe this problem is inapproximable also…

Conclusion

- Introduced a novel model, conditional scheduling with varying deadlines.
- Developed polynomial-time schedule synthesis algorithm for tree-shaped problems.
- Discussed computational hardness of discrete-time and DAG problems.

References

- [Baruah 1998a]
  S.K. Baruah. "Feasibility analysis of recurring branching tasks." EUROMICRO 1998.
- [Baruah1998b]
  S.K. Baruah. "A general model for recurring real-time tasks." RTSS 1998.
- [Chakraborty, Erlebach, and Thiele 2001]
  S. Chakraborty, T. Erlebach, L. Thiele. "On the complexity of scheduling conditional real-time code." WADS 2001.